AVIRAL CLASSES
IIT-JEE | NEET | FOUNDATIONS

ULTIMATE TEST SERIES JEE MAIN -2020
TEST-01 ANSWER KEY
Test Date :01-03-2020

[PHYSICS]

1. B
2. C
3. D

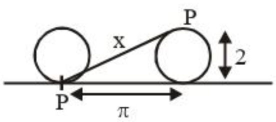
4. $a = v\dfrac{dv}{dx}$

5. $P = \sqrt{2mKE} \Rightarrow P \propto \sqrt{m}$

Momentum $\propto \sqrt{\text{mass}}$

mass $\downarrow$ momentum $\downarrow$

6. A

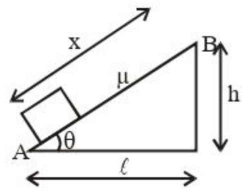
7. 

$x = \sqrt{\pi^2 + 4}$

8. m = linear density $= \dfrac{M}{L}$

$[B] = \left[\dfrac{A}{m}\right] = \left[\dfrac{F}{M/L}\right] = \left[\dfrac{FL}{M}\right]$ = dimensions of latent heat

9. A
10. C
11. B
12. C

13. 

$mg\sin\theta + \mu\, mg\cos\theta)x$

$Mg\left(\dfrac{h}{x} + u\dfrac{1}{x}\right).x$

$Mg\,(h + \mu\ell)$

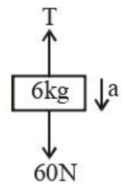
14. V = slope of x - t graph

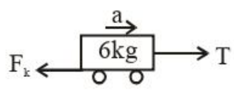
If sign of v changes, then direction reverses.

if $v\uparrow$, then $a > 0$ and if $v\downarrow$, then $a < 0$

15. D

16.





$60 - T = 6a$ .....(i)

$T = f_K = 30a$

$T - 30 \times 0.1 \times 10 = 30a$

$T - 30 = 5(6a)$

$T - 30 = 5\,(60 - T)$ (by eq. i)

$T - 30 = 300 - 5T$

$6T = 330$

$T = 55$ N

17. Impulse $= \Delta p = m\,(v_f - v_i)$

$= 0.5\left[-\dfrac{10}{5} - \dfrac{10}{5}\right]$

18.
$$KE_f = \frac{1}{4}\ KE_i$$
$$\frac{1}{2}\ mV^2 = \frac{1}{4}\left(\frac{1}{2}mV_0^2\right)$$
$$V = \frac{V_0}{2}$$
$$V = u + at \quad (a = \mu g)$$
$$\frac{V_0}{2} = V_0 - \mu g t_0$$
$$\mu g\ t_o = \frac{V_0}{2}$$
$$\mu = \frac{V_0}{2gt_0}$$

19. $\frac{4m_1m_2}{(m_1+m_2)^2}$

20. If length AB = x

$(mg \sin\theta + \mu mg \cos\theta)x$

$mgx\left(\frac{h}{x}+\frac{\mu\ell}{x}\right)$

INTEGER

21. **6**
22. **2**
23. **3**
24. **3**
25. **4**

[CHEMISTRY]

26. **A**

27. $BeSO_4 > MgSO_4 > CaSO_4 > SrSO_4 > BaSO_4$ (Solubility)

28. **A**
29. **B**
30. **A**
31. **A**
32. **A**
33. **B**
34. **D**
35. **D**
36. **C**
37. **D**
38. **B**
39. **A**
40. **D**
41. **A**
42. **D**
43. **A**
44. **C**
45. **C**

INTEGER

46. **9**

47.

$$2H_2 + O_2 \longrightarrow 2H_2O$$
Initial 8gm 16gm

32 gm of O_2 – 4gm H_2

16 gm of O_2 – $\frac{4}{32}\times 16 = 2$ gm

Amount of H_2 left = 6 gm

48. **1**
49. **3**
50. **2**

[MATHEMATICS]

51 B maximum value of cos (tan x) = 1
so Max. value of sin (cos (tan x)) = sin 1

52 **B** a, b are roots of cos b of $x^2 - 2x + 4 = 0$

$$x = \frac{2 \pm \sqrt{4-16}}{2}$$

$\alpha = 1 + \sqrt{3}\ i$

$x = 1 \pm 2\sqrt{3}\ i$

$\alpha = 1 + \sqrt{3}\ i$ $\beta - 1 - \sqrt{3}\ i$

$\alpha = 2.[\cos/3 + i\sin/3]'$ $\beta = 2.[\cos/3 - i\sin)_3]$

$a^n + b^n = 2^{n+1} . \cos\left(\frac{n\pi}{3}\right)$

53 **Ans. (2)**

$$\text{LHS}: \frac{\cos\frac{x}{3}}{\sin\frac{2x}{3}\cos\frac{x}{3}} = \text{cosec}\frac{2x}{3} \Rightarrow k = 2$$

$\tan^{-1}(\tan 2) = 2 - \pi.$

54 **C** $\sqrt{x+3-4\sqrt{x-1}} + \sqrt{x+8-6\sqrt{x-1}} = 1$

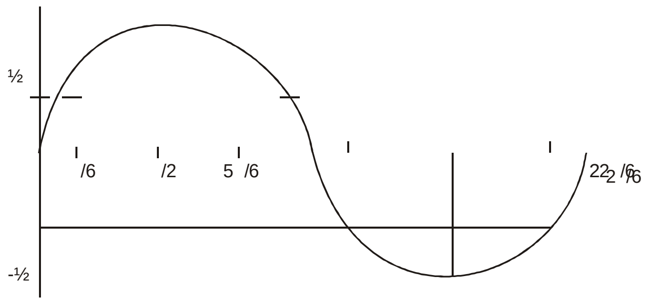
$\sqrt{(x-1)+4-4\sqrt{x-1}}$

$\sqrt{(x-2)^2} + \sqrt{(\sqrt{x-1}-3)^2} = 1$

$\left|\sqrt{x-1}-2\right| + \left|\sqrt{x-1}-3\right| = 1$

55 **D** $2\cos^2\theta + \sin\theta \le 2$

$\sin\theta(2\sin\theta - 1) \ge 0$



Y = sin

so $x \in \left(\frac{\pi}{2}, \frac{5\pi}{6}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

56. **Ans. (3)**

$\because$ a = 0 and y = $bx^2 + cx + d$ is symmetric about $x = -\frac{c}{2b}$

$\therefore x = k = -\frac{c}{2b} \Rightarrow k + \frac{c}{2b} = 0$

$\Rightarrow a + \frac{c}{2b} + k = 0$

57. **Ans. (1)**

$$\frac{1}{2} + \frac{1}{2\sin\frac{x}{2}} 2\sin\frac{x}{2}(\cos x + \cos 2x + \cos 3x + \cos 4x) = 0$$

$$= \frac{1}{2} + \frac{1}{2\sin\frac{x}{2}}\left(\sin\frac{9x}{2} - \sin\frac{x}{2}\right) = 0$$

$$= \frac{\sin\left(\frac{9x}{2}\right)}{\sin\left(\frac{x}{2}\right)} = 0 \Rightarrow x = \frac{2n\pi}{9}, n \ne 9m\ m \in I$$

58. **Ans. (3)**

$$\cot x = \frac{1}{2}\left(\cot\frac{x}{2} - \tan\frac{x}{2}\right)$$

$$\cot x = \frac{1}{2}\left\{\frac{1}{2}\left(\cot\frac{x}{4} - \tan\frac{x}{4}\right) - \tan\frac{x}{2}\right\}$$

$$= \frac{1}{4}\cot\frac{x}{4} - \frac{1}{4}\tan\frac{x}{4} - \frac{1}{2}\tan\frac{x}{2}$$

$$= \frac{1}{8}\left(\cot\frac{x}{8} - \tan\frac{x}{8}\right) - \frac{1}{4}\tan\frac{x}{4} - \frac{1}{2}\tan\frac{x}{2}$$

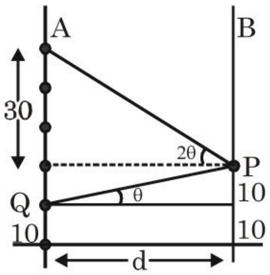
59. **Ans. (2)**

$$\Delta = \frac{1}{2}ah_1 = \frac{1}{2}bh_2 = \frac{1}{2}ch_3$$

$$h_1 = \frac{2\Delta}{a} \text{ and } h_2 = \frac{2\Delta}{b} \text{ and } h_3 = \frac{2\Delta}{c}$$

$$\frac{1}{h_1} + \frac{1}{h_2} - \frac{1}{h_3} = \frac{1}{2\Delta}(a + b - c) = \frac{2\sqrt{7}}{15}$$

60. **Ans. (1)**

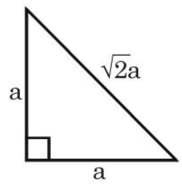


$d = 10\cot\theta$; $d = 30\cot 2\theta$

$10\cot\theta = 3\cot 2\theta$

$\Rightarrow \theta = 30^\circ$

61. **Ans. (1)**

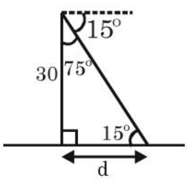


$$r = \frac{\frac{1}{2}.a^2}{a + \frac{a}{\sqrt{2}}} \Rightarrow 1 = \frac{a}{2+\sqrt{2}}$$

$$\Delta = \frac{1}{2}a^2 = \frac{1}{2}.\left(4+2+4\sqrt{2}\right)$$

$$= 3 + 2\sqrt{2}$$

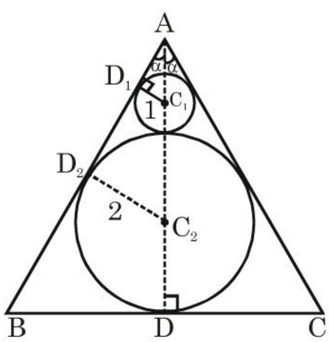
62. **Ans. (3)**



$$\tan 15° = \frac{30}{d}$$

$$d = \frac{30}{2-\sqrt{3}} = \frac{30.\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)}$$

63. **Ans. (1)**



$$\sin\alpha = \frac{1}{3}$$

$\therefore AC_1 = 3$

$AC_2 = 6$

$AD = 8$

$\therefore BD = 2\sqrt{2}$

$$\text{Area} = \frac{1}{2}.4\sqrt{2}.8 = 16\sqrt{2}$$

64. **Ans. (3)**

$y = mx + 1$ is tangent to ellipse

$x^2 + 4y^2 = 1$ in Ist quadrant $\therefore m < 0$

$$\therefore 1 = m^2 + \frac{1}{4}$$

$$m = \frac{\sqrt{3}}{2} \text{ or } -\frac{\sqrt{3}}{2}$$

(reject)

65. **Ans. (3)**

Given $2b = a + c \Rightarrow \frac{2b}{a} = 1 + \frac{c}{a}$ ...(i)

$$\alpha + \beta = -\frac{b}{a} = 15 \Rightarrow \frac{b}{a} = -15 \Rightarrow \frac{c}{a} = -31$$

$\alpha\beta = -31.$

66. **Ans. (3)**

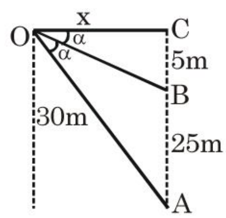
$(\tan^2 x - 1)^2 = 3 - [a]^2$

Hence, $3 - [a]^2 \geq 0 \Rightarrow [a] \in \left[-\sqrt{3}, \sqrt{3}\right]$

$\therefore [a] = -1, 0, 1$

$\Rightarrow a \in [-1, 2)$

67. **Ans. (2)**



$$\tan\alpha = \frac{5}{x} \quad \text{...(i)}$$

$$\tan 2\alpha = \frac{30}{x} \quad \text{...(ii)}$$

from (i) and (ii) $\tan\alpha = \sqrt{\frac{2}{3}}$

$$\therefore x = 5\cot\alpha = 5\sqrt{\frac{3}{2}}$$

68. $F : N \to N$ $f(x) = 2x + 3$

as a linear function f is on-one but range is not all so one-one into

69. **Ans. (4)**

Reflexive, symmetric but not transitive.

70. **Ans. (2)**

$$\left.\begin{array}{l} f(1) \le 0 \\ f(2) \le 0 \end{array}\right\} \cap$$

$f(1) = 1 - 2a + a^2 - 6a \le 0$

$a^2 - 8a + 1 \le 0 \Rightarrow a \in \left[4 - \sqrt{15}, 4 + \sqrt{15}\right]$ ...(i)

$f(2) = 4 - 4a + a^2 - 6a \le 0$

$a^2 - 10a + 4 \le 0$

$a \in \left[5 - \sqrt{21}, 5 + \sqrt{21}\right]$ ...(ii)

$(1) \cap (2) \Rightarrow a \in \left[5 - \sqrt{21}, 4 + \sqrt{15}\right]$

INTEGER

71. $f(x) = 2 + \dfrac{3}{x^4 - 7x^2 - 4x + 23}$

Let $h(x) = x^4 - 7x^2 - 4x + 23$

$= \left(x^2 - 4\right)^2 + \left(x - 2\right)^2 + 3$

$h(x) \ge 3$

Range of $h(x)$ is $[3, \infty)$

$\Rightarrow$ Range of $f(x)$ is $(2, 3]$

72. $x^2 - \sqrt{2}x + 1 = 0$

$\therefore\ \alpha = \dfrac{1}{\sqrt{2}} + i\dfrac{1}{\sqrt{2}}, \beta = \dfrac{1}{\sqrt{2}} - i\dfrac{1}{\sqrt{2}}$

$= e^{i\pi/4}$ $\quad = e^{-i\pi/4}$

$\alpha^{50} + \beta^{50} = e^{i25\pi/2} + e^{-i25\pi/2} = i + (-i) = 0$

73. $f(x) = f(-x)\ \forall\ x \in R$

$\therefore\ 2Ax^3 - 2Bx = 0\ \forall x \in R \Rightarrow A = B = 0$

simultaneously

$A = 0 \Rightarrow \left(\sin\alpha - \dfrac{1}{2}\right)^2 = 0 \Rightarrow \sin\alpha = \dfrac{1}{2}$

$B = 0 \Rightarrow \left(\tan\alpha + \dfrac{1}{\sqrt{3}}\right)^2 = 0 \Rightarrow \tan\alpha = -\dfrac{1}{\sqrt{3}}$

Hence 2 solution only.

74. $\sin 5\theta \cos 3\theta = \sin 9\theta \cos 7\theta$

using $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$

$\sin 8\theta = \sin 16\theta$

$\sin 16\theta - \sin 8\theta = 0$

$\sin 8\theta(2 \cos 8\theta - 1) = 0$

$\sin 8\theta = 0$

$\cos 8\theta = 1/2$

so no. of solutions in $[0, \pi/4] = 5$

75. **4** Difference roots of equation

$| a - \beta) = |\beta^1 - \beta^1|$

$\sqrt{(\alpha + \beta)^2 - 4\alpha\beta} = \sqrt{(\alpha + \beta)^2 - 4\,\alpha\beta}$'

$\sqrt{(a^2 - 4b)} = \sqrt{b^2 - 4a}$

$(a^2 - b^2) + (4a - 4b) = 0$

$(a + b)(a - b) + 4(a - b) = 0$

$(a - b)(a + b + 4) = 0$

$-| a + b | = 4$